Quiz 3, Calculus I - No calculators

Dr. Graham-Squire, Fall 2017

4 min ⇒ 15 → 20 min in class

Name: Key

1. (4 points) Find y' for the equation

$$\frac{d}{dx}\left(y^2 + \arctan(x) = (\ln|x|)(\sin y) \right)$$

$$\Rightarrow \quad 2y\,y' + \frac{1}{1+x^2} = \frac{1}{x}\cdot \sin y + \ln|x| \cdot \cos y \cdot y'$$

$$2y\,y' - (\ln|x|)(\cos y)\,y' = \frac{\sin y}{x} - \frac{1}{1+x^2}$$

$$(2y - (\ln|x|)(\cos y))\,y' = \frac{\sin y}{x} - \frac{1}{1+x^2}$$

$$y' = \frac{\left(\frac{\sin y}{x} - \frac{1}{1+x^2}\right)}{2y - (\ln|x|)(\cos y)}$$

2. (3 points) Calculate the derivative below. You do NOT need to simplify.

$$f(x) = \left(5x^2 - \sqrt{6^x + x^8}\right)^4$$

$$f'(x) = 4\left(5x^2 - (6^x + x^8)^{1/2}\right)^3 \cdot \left[10x - \frac{1}{2}(6^x + x^8)^{-1/2} \cdot (6^x \ln 6 + 8x^7)\right]$$

3. (3 points) Calculate the derivative of $h(x)$. You can either simplify the expression using logarithm rules, and then take the derivative, OR take the derivative directly and then simplify.

$$h(x) = \ln\left(\frac{(e^x)(\cos x)}{(x+7)^6}\right)^2$$

1.5 for deriv.
1.5 for simplificata.

$$h(x) = 2\ln\left(\frac{e^x(\cos x)}{(x+7)^6}\right)$$

$$= 2\left(\ln(e^x) + \ln(\cos x) - \ln(x+7)^6\right)$$

$$h(x) = 2\left(x + \ln(\cos x) - 6\ln(x+7)\right)$$

$$h'(x) = 2\left(1 + \frac{1}{\cos x}\cdot(-\sin x) - 6\left(\frac{1}{x+7}\cdot 1\right)\right)$$

$$= \boxed{2\left(1 - \frac{\sin x}{\cos x} - \frac{6}{x+7}\right)}$$

or $2 - 2\tan x - \frac{12}{x+7}$

or $h'(x) =$

$$\frac{1}{\left(\frac{e^x\cos x}{(x+7)^6}\right)^2} \cdot 2\left(\frac{e^x(\cos x)}{(x+7)^6}\right) \cdot \frac{(x+7)^6(e^x\cos x + e^x(-\sin x)) - e^x\cos x \cdot 6(x+7)^5}{(x+7)^{12}}$$

$$= \frac{2}{\left(\frac{e^x\cos x}{(x+7)^6}\right)} \cdot \frac{e^x(x+7)^5\left[(x+7)(\cos x - \sin x) - 6\cos x\right]}{(x+7)^{12}}$$

$$= 2(x+7)^6 e^x (x+7)^5 (\ldots)$$

$$= \frac{2\left[(x+7)(\cos x - \sin x) - 6\cos x\right]}{(\cos x)(x+7)}$$